

# NLO evolution kernels for skewed transversity distributions.

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## Abstract

We present a calculation of the two-loop evolution kernels of the twist-two transversally polarized skewed quark and linearly polarized skewed gluon distributions in the minimal subtraction scheme and discuss a solution of the evolution equations suitable for numerical implementation.

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# 1 Introduction.

Helicity-flip parton densities, i.e. densities of transversally polarized quarks and linearly polarized gluons, are inaccessible in conventional deep inelastic scattering experiments on polarized spin- $\frac{1}{2}$  targets. The transversally polarized quark density [1] is of odd chirality, and thus requires a helicity flip in a perturbative QCD subprocess which is forbidden for massless quarks, while the linearly polarized gluon requires the helicity to be flipped by two units which is not allowed by angular momentum conservation for spin- $\frac{1}{2}$  hadrons. However, it will definitely appear for spin- $J \geq 1$  hadrons/photon [2, 3].

If one goes from the forward to off-forward kinematics, i.e. inclusive DIS to exclusive DVCS with a real final state photon [4, 5, 6], one allows for non-zero orbital momentum in the system and thus the gluons can transfer two units of helicity from photons to spin- $\frac{1}{2}$  hadrons [7, 8, 9]. Due to specific  $\cos / \sin(3\phi)$  azimuthal angle dependences in the factorized [6, 10, 11] cross section of the corresponding skewed tensor gluon distribution (SPD), it can be extracted [9] without the usual complication due to quark contamination. A similar possibility exists for a system of hadrons due to their orbital motion, as it occurs in the  $\gamma\gamma \rightarrow \pi\pi$  reaction [12]. As to the quark sector, unfortunately, the original proposal to measure the skewed quark transversity in diffractive meson production [13] has been proven erroneous due to the preservation of chiral symmetry for the cross section [14] and thus complicates the experimental access to the function in question.

Note, however, in DVCS, since the photon helicity flip requires a change in the hadron helicity by two units, the amplitude on a proton target is suppressed (due to its spin being only  $\frac{1}{2}$ ) by an extra power of the transverse momentum transfer in the  $t$ -channel  $\Delta_\perp$  whereas it will not be suppressed on a deuteron target, which is available at certain facilities like HERMES. Given its current run on high density targets, it is very interesting that high statistics data will be shortly forthcoming and hence there is a need to study the helicity-flip amplitudes theoretically.

In this paper, we present an evaluation of the two-loop skewed evolution kernels for chiral odd quark and tensor gluon distributions. Since corresponding operators which define the non-perturbative content of the former belong to different representations of the Lorentz group the mixing is absent in both cases. Our presentations is structured as follows. In the second section, we discuss the general solution of the skewed evolution equations, in the third one, we present the reconstruction of the skewed evolution kernels in the Efremov-Radyushkin-Brodsky-Lepage (ER-BL) and Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) regions, and finally, we summarize.

## 2 Evolution equation.

SPDs [4]-[6, 13] appear in various exclusive processes and their extraction would provide us new, valuable, non-perturbative information on the structure of hadrons. These functions can be considered as a link between Feynman's parton densities and exclusive quantities like form factors/distribution amplitudes, in fact they are generalizations of both. In QCD they are defined as expectation values of twist-two light-ray operators over the hadronic states,

$$\left\{ \begin{matrix} Qq^T \\ Gq^T \end{matrix} \right\} (x, \eta, \Delta^2; Q^2) = \left\{ \begin{matrix} 1 \\ 4P_+^{-1} \end{matrix} \right\} \int \frac{d\kappa}{2\pi} e^{i\kappa x P_+} \langle P_2 | \left\{ \begin{matrix} Q\mathcal{O}^T \\ G\mathcal{O}^T \end{matrix} \right\} (\kappa, -\kappa)_{|\mu^2=Q^2} | P_1 \rangle, \quad (1)$$

where  $P_+ = n \cdot (P_1 + P_2)$  with  $n_\mu$  being a light cone vector. These are functions of the momentum fraction  $x$ , the longitudinal momentum fraction  $\eta > 0$  in the  $t$  channel, called the skewedness parameter, the momentum transfer square  $\Delta^2 = (P_2 - P_1)^2$ , and they vary depending on the resolution scale  $Q^2$ . The partonic interpretation of these functions depends on the kinematical region. For the ER-BL region  $|x| < \eta$  they behave as distribution amplitudes of meson-like states, while for the DGLAP domain  $|x| > \eta$  they mimic the conventional probability distribution of finding a parton with a corresponding momentum and spin state that is scattered from the initial to the final state.

Now we recall the general properties of the evolution equation for SPDs,

$$\frac{d}{d \ln Q^2} Aq^T(x, \eta, \Delta^2; Q^2) = \int_{-1}^1 \frac{dy}{2|\eta|} {}^{AA}V^T \left( \frac{\eta+x}{2\eta}, \frac{\eta+y}{2\eta}; \alpha_s(Q^2) \right) Aq^T(y, \eta, \Delta^2; Q^2), \quad (2)$$

with the kernel  ${}^{AA}V^T$  defined as a series in the coupling  $\alpha_s$ . Note, that we treat both quark and gluon cases simultaneously and that there is no mixing between them, as mentioned in the introduction, since they belong to different representations of the Lorentz group.

The evolution kernel  ${}^{AA}V^T(x, y)$  with  $0 \leq x, y \leq 1$  is of the ER-BL-type and governs the evolution of meson distribution amplitudes (formally  $|\eta| \equiv 1$ ). The general structure of this kernel was already studied in the past [4] with the main result being that the evolution kernels of skewed parton distributions can be obtained from the ER-BL ones,

$${}^{AA}V^T = \theta(y-x)f(x, y) + \theta(y-\bar{x})g(x, y) + \left\{ \begin{matrix} x \rightarrow \bar{x} \\ y \rightarrow \bar{y} \end{matrix} \right\}, \quad \bar{x} \equiv 1-x, \quad 0 \leq x, y \leq 1, \quad (3)$$

[here  $x$  denotes a new variable, which differs from that one used in Eq. (2)] by replacing the  $\theta$  structure:

$$\theta(y-x) \rightarrow \Theta \left( \frac{\eta+x}{2\eta}, \frac{\eta+y}{2\eta} \right) \equiv \theta \left( 1 - \frac{\eta+x}{\eta+y} \right) \theta \left( \frac{\eta+x}{\eta+y} \right) \text{sign} \left( \frac{\eta+y}{\eta} \right), \quad (4)$$

for  $x \rightarrow \frac{1}{2\eta}(\eta+x)$ , and analytical continuation of the prefactors  $f$  and  $g$ . In the gluonic case the second  $\theta$  structure can be reduced to the first one by means of Bose symmetry, namely,

$G_q^T(x, \eta) = G_q^T(-x, \eta)$ , while in the quark case, we should decompose the SPD in contributions<sup>1</sup> coming from quarks ( $x \geq 0$ ) and anti-quarks ( $x \leq 0$ ).

The evolution equation (2) can be solved by means of different methods. However, most of them are plagued by either a limitation of their applicability to a leading order (LO) analysis or a lack of sufficient accuracy in handling particular regions of the phase space. Only the orthogonal polynomial reconstruction method [15, 16] and the direct numerical integration methods [17, 18] allow for a generalization beyond the one-loop approximation. However, the first method suffers from numerical complications in the treatment of the transition region  $|x| = \eta$  and, therefore, one may explore the efficiency of a direct numerical integration to overcome this particular difficulty. To do so we should separate the evolution equation into DGLAP and ER-BL regions.

In the quark sector, it proves convenient to introduce (non-) singlet like combinations of SPDs that are symmetric/antisymmetric w.r.t. the momentum fraction  $x$ :

$$Q_q^T(x, \eta) = \frac{1}{2} \left[ {}^+q^T(x, \eta) - {}^-q^T(-x, \eta) \right], \quad {}^\pm q^T(x, \eta) = Q_q^T(x, \eta) \mp Q_q^T(-x, \eta). \quad (5)$$

Here the antiquark contribution is given by  $-Q_q^T(-x, \eta)$  and thus  ${}^\pm q^T(x, \eta)$  for  $x > 0$  is just the sum and difference of quark and anti-quark distributions, respectively. The kernels in the corresponding evolution equations read

$$\begin{aligned} {}^{QQ}V^{T\pm} \left( \frac{\eta+x}{2\eta}, \frac{\eta+y}{2\eta} \right) &= \Theta \left( \frac{\eta+x}{2\eta}, \frac{\eta+y}{2\eta} \right) {}^{QQ}F^{T\pm} \left( \frac{\eta+x}{2\eta}, \frac{\eta+y}{2\eta} \right) + \left\{ \begin{array}{l} x \rightarrow -x \\ y \rightarrow -y \end{array} \right\}, \\ {}^{QQ}F^{T\pm}(x, y) &= {}^{QQ}f^T(x, y) \mp {}^{QQ}g^T(\bar{x}, y). \end{aligned} \quad (6)$$

Based on the support and symmetry properties, we discuss now the solution of this evolution equation (2). In the following  ${}^\pm q(x, \eta, Q^2)$  is a (anti-) symmetric skewed distribution of any species, where for gluons we formally have to take the solution for  ${}^-q(x, \eta, Q^2)$ . An analogous sign convention holds true for unpolarized partons, while for longitudinal polarized ones, one should replace  $+ \leftrightarrow -$ . In the DGLAP region we have a homogeneous integro-differential equation,

$$\frac{d}{d \ln Q^2} {}^\pm q(x, \eta; Q^2)_{|x| > \eta} = \int_x^1 \frac{dy}{2|\eta|} \left( {}^\pm F - {}^\pm \bar{F} \right) \left( \frac{\eta+x}{2\eta}, \frac{\eta+y}{2\eta} \right) {}^\pm q(y, \eta; Q^2), \quad (7)$$

while in the ER-BL region an inhomogeneous term arises,

$$\begin{aligned} \frac{d}{d \ln Q^2} {}^\pm q(x, \eta; Q^2)_{|x| < \eta} &= \int_{-|\eta|}^{|\eta|} \frac{dy}{2|\eta|} {}^\pm V \left( \frac{\eta+x}{2\eta}, \frac{\eta+y}{2\eta} \right) {}^\pm q(y, \eta; Q^2) + {}^\pm I(x, \eta; Q^2), \\ {}^\pm I(x, \eta; Q^2) &= \int_{|\eta|}^1 \frac{dy}{2|\eta|} \left[ {}^\pm F \left( \frac{\eta+x}{2\eta}, \frac{\eta+y}{2\eta} \right) \mp {}^\pm \bar{F} \left( \frac{\eta-x}{2\eta}, \frac{\eta+y}{2\eta} \right) \right] {}^\pm q(y, \eta; Q^2), \end{aligned} \quad (8)$$

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<sup>1</sup>More precisely, we should speak of (anti) quark distributions only for  $x \geq \eta$  and ( $x \leq -\eta$ ), while there are two meson-like distributions with different symmetry in the ER-BL region.

where we used the short-hand notation  $\overline{F}(x, y) \equiv F(\bar{x}, \bar{y})$ . Introducing the kernel  $P\left(\frac{x}{y}, \frac{\eta}{y}\right) = \frac{y}{|\eta|} \left(F - \overline{F}\right)\left(\frac{\eta+x}{2\eta}, \frac{\eta+y}{2\eta}\right)$ , it is obvious that equation (7) is of the DGLAP type. Since, in addition, the kernel depends on the skewedness parameter  $\eta$ , there does not exist the same factorization for the Mellin moments as in the forward case. Fortunately, this equation can be efficiently integrated by brute force [17]. We formally write the solution of this equation as a convolution of the skewed distribution given at the input scale  $Q_0^2$  with an evolution operator

$$q(x, \eta; Q^2)_{|x|>\eta} = \int_{|\eta|}^1 \frac{dy}{y} U_{\text{DGLAP}}(x, y, \eta; Q^2, Q_0^2) q(y, \eta; Q_0^2), \quad (9)$$

that satisfies the equation

$$\frac{d}{d \ln Q^2} U_{\text{DGLAP}}(x, z, \eta; Q^2, Q_0^2) = \int_x^1 \frac{dy}{y} P\left(\frac{x}{y}, \frac{\eta}{y}\right) U_{\text{DGLAP}}(y, z, \eta; Q^2, Q_0^2) \quad (10)$$

with the initial condition  $U_{\text{DGLAP}}(x, y, \eta; Q_0^2, Q_0^2) = \delta\left(1 - \frac{x}{y}\right)$ . The homogeneous part of the evolution equation (8) is precisely of the ER-BL type and its solution is

$$q^{\text{hom}}(x, \eta; Q^2)_{|x|<\eta} = \int_{-1}^1 dy U_{\text{ERBL}}\left(\frac{x}{\eta}, y; Q^2, Q_0^2\right) q^{\text{hom}}(\eta y, \eta; Q_0^2), \quad (11)$$

with  $U_{\text{ERBL}}(x, y, Q_0^2, Q_0^2) = \delta(x - y)$ . A particular solution for the inhomogeneous equation is easily constructed by means of the inverse evolution operator and we obtain, together with the homogeneous part,

$$\begin{aligned} q(x, \eta; Q^2)_{|x|<\eta} = & \int_{-1}^1 dy U_{\text{ERBL}}\left(\frac{x}{\eta}, y; Q^2, Q_0^2\right) \left\{ q(\eta y, \eta; Q_0^2) \right. \\ & \left. + \int_{-1}^1 dz \int_{Q_0^2}^{Q^2} \frac{dQ'^2}{Q'^2} U_{\text{ERBL}}^{-1}(y, z; Q'^2, Q_0^2) I(\eta z, \eta; Q'^2) \right\}, \end{aligned} \quad (12)$$

where  $I(x, \eta; Q'^2)$  is the convolution of Eq. (8) with the solution of Eq. (9) in the DGLAP region. An important issue is to study the evolution at the point  $|x| = \eta$ . Since we have the property  $F(x = 0, y) = \overline{F}(x = 1, y) = 0$  in both LO and next-to-leading (NLO) [see below Eqs. (18), (39), and (40)], the limit  $\epsilon \rightarrow 0$  of the evolution equations for  $x = \eta + \epsilon$  and  $x = \eta - \epsilon$  will be the same. Therefore, one expects that a smooth input function will remain smooth under evolution. Nevertheless, this point necessitates care as it is a delicate point in the numerical solution of the evolution equations.

The construction of the ER-BL from the DGLAP kernels is in principle possible, provided we know the eigenfunctions of the former. In LO the exclusive kernels are diagonal w.r.t. Gegenbauer polynomials

$$\int_{-1}^1 \frac{dx}{2|\eta|} C_{j+3/2-\nu(A)}^{\nu(A)} \left(\frac{x}{\eta}\right)^{AA} V^{(0)T} \left(\frac{\eta+x}{2\eta}, \frac{\eta+y}{2\eta}\right) = -\frac{1}{2} {}^{AA} \gamma_j^{(0)T} C_{j+3/2-\nu(A)}^{\nu(A)} \left(\frac{y}{\eta}\right), \quad (13)$$

where  $\nu(A) = \left\{\frac{3}{2}, \frac{5}{2}\right\}$  for  $A = \{Q, G\}$  and  ${}^{AA}\gamma_j^{(0)T}$  are the forward anomalous dimensions at LO. This fact reflects the underlying conformal symmetry at tree-level. Consequently, this symmetry allows us to reconstruct the ER-BL kernel from the DGLAP kernel by an integral transformation [for details see second article of [19]],  ${}^{AA}V^{(0)T}(x, y) = \int_0^1 dz G(2x - 1, 2y - 1; z|\nu(A)) {}^{AA}P^{(0)T}(z)$ , where the integral kernel is given in terms of a hypergeometric function

$$G(x, y; z|\nu) = \frac{\Gamma(\nu)\Gamma(\nu+1)}{\Gamma^2(\frac{1}{2})\Gamma(2\nu)} \frac{2^{2\nu}(1-x^2)^{\nu-1/2}(1-z^2)}{\left[1 - 2\left(xy - \sqrt{(1-x^2)(1-y^2)}\right)z + z^2\right]^{\nu+1}} \quad (14)$$

$$\times {}_2F_1\left(\begin{matrix} \nu+1, \nu \\ 2\nu \end{matrix} \middle| \frac{4\sqrt{(1-x^2)(1-y^2)}z}{1 - 2\left(xy - \sqrt{(1-x^2)(1-y^2)}\right)z + z^2}\right).$$

Note that, in any order of perturbation theory, the evolution operator in the ER-BL region can be represented in terms of an infinite series of Gegenbauer polynomials [23]. Unfortunately, since in general a SPD does not vanish at the point  $x = \pm\eta$ , it is expected that, to achieve a good approximation, a large number of terms in this expansion is needed. Fortunately, we can resum the Gegenbauer expansion in the ER-BL region which provides us the integral transformation, mentioned above, of the evolution operator in the forward region

$$U_{\text{ERBL}}(x, y; Q^2, Q_0^2) = \frac{1}{2} \int_0^1 dz G(x, y, z|\nu(A)) U_{\text{DGLAP}}(z, z' = 1, \eta = 0; Q^2, Q_0^2), \quad (15)$$

where  $G(x, y; z|\nu)$  is the kernel defined in Eq. (14). Beyond the LO approximation, one can perturbatively expand the solution with the help of this evolution operator. From the representation of this operator in terms of Gegenbauer polynomials it is obvious that the inverse operator is  $U_{\text{ERBL}}^{-1}(x, y; Q^2, Q_0^2) = U_{\text{ERBL}}(x, y; Q_0^2, Q^2)$ .

Let us add that in the forward case, i.e.  $\langle P_2 | \rightarrow \langle P_1 |$ , Eqs. (1) define the chiral odd quark density and the tensor gluon density times the momentum fraction  $z$ , thus, we have the limit of the splitting functions:

$$\left\{ \begin{matrix} QQP^T \\ GGP^T \end{matrix} \right\}(z) = \text{LIM} \left\{ \begin{matrix} QQV^T \\ GGVT \end{matrix} \right\}(x, y) \equiv \lim_{\eta \rightarrow 0} \frac{1}{|\eta|} \left\{ \begin{matrix} QQV^T \\ \frac{1}{z} GGV^T \end{matrix} \right\} \left( \frac{z}{\eta}, \frac{1}{\eta} \right), \quad (16)$$

which will be of relevance to the analysis in the following section.

### 3 Construction of two-loop kernels.

Since the extension of the ER-BL kernel to the whole region is unique [4], it is sufficient to construct the evolution kernels,

$${}^{AA}V^T(x, y) = \frac{\alpha_s}{2\pi} {}^{AA}V^{T(0)}(x, y) + \left(\frac{\alpha_s}{2\pi}\right)^2 {}^{AA}V^{T(1)}(x, y) + \mathcal{O}(\alpha_s^3), \quad (17)$$

in the ER-BL-region. The one loop kernels have been obtained by a direct calculation [21, 22, 8, 9]. Alternatively, they can be deduced by means of conformal covariance (13) from the forward kernels via the integral transformation (14):

$$\left\{ \begin{matrix} QQ \\ GG \end{matrix} V^{T(0)} \right\} = \left\{ \begin{matrix} C_F \\ C_A \end{matrix} \right\} \left[ \theta(y-x) \left\{ \begin{matrix} QQ \\ GG \end{matrix} f^T \right\} + \left\{ \begin{matrix} x \rightarrow \bar{x} \\ y \rightarrow \bar{y} \end{matrix} \right\} \right]_+ - \frac{1}{2} \left\{ \begin{matrix} QQ \\ GG \end{matrix} \gamma_0^{T(0)} \right\} \delta(x-y), \quad (18)$$

where

$$QQf^T \equiv QQf^b = \frac{x}{y} \frac{1}{y-x}, \quad QQ\gamma_0^{T(0)} = C_F, \quad GGf^T \equiv GGf^b = \frac{x^2}{y^2} \frac{1}{y-x}, \quad GG\gamma_1^{T(0)} = 6C_A + \beta_0. \quad (19)$$

Here the  $+$ -prescription is defined for both channels in the same way, namely as  $[V(x, y)]_+ = V(x, y) - \delta(x-y) \int_{-1}^1 dz V(z, y)$ . As explained in great detail in [19], in NLO conformal covariance does not hold true anymore for the minimal subtraction (MS) scheme in the dimensional regularized theory. Besides a symmetry breaking term proportional to the QCD  $\beta$ -function, there appears an additional term which is induced by the leading order anomaly in the special conformal transformations of conformal operators. Note, that the latter anomalies are renormalization scheme dependent even in one-loop approximation. Next, the results for the local conformal operators in the MS scheme can be transformed to the kernels, which have the following structure in NLO:

$$AAV^{T(1)} = -AA\dot{V}^T \otimes \left( AA V^{T(0)} + \frac{\beta_0}{2} \mathbb{1} \right) - \left[ AA g^T \otimes AA V^{T(0)} \right] + AA \mathcal{D}^T, \quad (20)$$

where the commutator stands for  $[A \otimes B](x, y) = \int_0^1 dz \{A(x, z)B(z, y) - B(x, z)A(z, y)\}$ . The off-diagonal part w.r.t. Gegenbauer polynomials is contained in the first two convolutions on the r.h.s. of this equation. The third term, i.e.  $AA \mathcal{D}^T$ , is diagonal w.r.t. Gegenbauer polynomials with index  $\nu(A) = \left\{ \frac{3}{2}, \frac{5}{2} \right\}$  for  $A = \{Q, G\}$ . The dotted kernel is derived from the LO ones via a logarithmic modification,

$$QQ\dot{V}^{T(0)} = C_F \theta(y-x) \frac{x}{y} \frac{1}{y-x} \ln \frac{x}{y} + \left\{ \begin{matrix} x \rightarrow \bar{x} \\ y \rightarrow \bar{y} \end{matrix} \right\}, \quad (21)$$

$$GG\dot{V}^{T(0)} = C_A \theta(y-x) \frac{x^2}{y^2} \frac{1}{y-x} \ln \frac{x}{y} + \left\{ \begin{matrix} x \rightarrow \bar{x} \\ y \rightarrow \bar{y} \end{matrix} \right\}. \quad (22)$$

The  $g$ -kernels, meanwhile, contain new information and are defined by

$$QQg^T = -C_F \left[ \theta(y-x) \frac{\ln \left( 1 - \frac{x}{y} \right)}{y-x} + \left\{ \begin{matrix} x \rightarrow \bar{x} \\ y \rightarrow \bar{y} \end{matrix} \right\} \right]_+, \quad (23)$$

$$GGg^T = -C_A \left[ \theta(y-x) \left( \frac{\ln \left( 1 - \frac{x}{y} \right)}{y-x} - 2 \frac{x}{y} \right) + \left\{ \begin{matrix} x \rightarrow \bar{x} \\ y \rightarrow \bar{y} \end{matrix} \right\} \right]_+. \quad (24)$$

In contrast to the  $QQ$ -kernel, which is unique in all sectors [19], the  $GG$ -kernel differs from those in the chiral even case by the term  $2C_A \frac{x}{y}$  [9].

Since it is difficult to project onto the off-diagonal part and apply the integral transformation to the NLO DGLAP kernel [20], we use another strategy for the construction of the remaining diagonal term,  $\mathcal{D}$ . This term is decomposed into a contribution related to the crossed-ladder diagram containing dilogarithms,  $G$ , and a remainder  $D$  which is, for the present helicity-flip sector, given as a linear combination of one-loop kernels,

$$\left\{ \begin{matrix} QQ \\ GG \end{matrix} \mathcal{D}^T \right\} (x, y) = -\frac{1}{2} \left\{ \begin{matrix} C_F \left( C_F - \frac{C_A}{2} \right) [{}^{QQ}G^T(x, y)]_+ \\ C_A^2 [{}^{GG}G^T(x, y)]_+ \end{matrix} \right\} + \left\{ \begin{matrix} QQ \\ GG \end{matrix} D^T \right\} (x, y). \quad (25)$$

In the following the  $G^T$  kernels are defined as

$${}^{AA}G^T(x, y) = \theta(y - x) \left( {}^{AA}h^T + \Delta {}^{AA}h^T \right) (x, y) + \theta(y - \bar{x}) \left( {}^{AA}\bar{h}^T + \Delta {}^{AA}\bar{h}^T \right) (x, y), \quad (26)$$

where

$$\begin{aligned} {}^{AA}h^T &= 2 {}^{AA}\bar{f}^T \ln \bar{x} \ln y - 2 {}^{AA}f^T [\text{Li}_2(x) + \text{Li}_2(\bar{y})], \\ {}^{AA}\bar{h}^T &= \left( {}^{AA}f^T - {}^{AA}\bar{f}^T \right) \left[ 2\text{Li}_2 \left( 1 - \frac{x}{y} \right) + \ln^2 y \right] + 2 {}^{AA}f^T [\text{Li}_2(\bar{y}) - \ln x \ln y] + 2 {}^{AA}\bar{f}^T \text{Li}_2(\bar{x}). \end{aligned} \quad (27)$$

Here the  $QQ$  kernel follows from the explicit flavor non-singlet two-loop result in the chiral even sector [24] by replacing  ${}^{QQ}f^{\text{NS}} \rightarrow {}^{QQ}f^T$ , with the addenda  $\Delta {}^{QQ}h^T$  and  $\Delta {}^{QQ}\bar{h}^T$  being zero. Note that we rewrote the original result in such a way that both  $\theta$ -contributions are symmetric with respect to the weight function  $x\bar{x}$ , i.e.  $y\bar{y}{}^{QQ}h^T(x, y) = x\bar{x}{}^{QQ}h^T(\bar{y}, \bar{x})$  and  $y\bar{y}{}^{QQ}\bar{h}^T(x, y) = x\bar{x}{}^{QQ}\bar{h}^T(y, x)$ . Now it can be verified that both parts are separately diagonal w.r.t. Gegenbauer polynomials.

Since the two-loop crossed-ladder diagram contains only a simple pole in the dimensional regularization parameter, the  $\mathcal{N} = 1$  supersymmetric constraints on the level of local operators lead to the equality of chiral odd quark and tensor gluon anomalous dimensions  ${}^{QQ}\gamma_j^{T+} = {}^{GG}\gamma_j^T$  [21, 25]. From this constraint one can derive a relation between the momentum fraction kernels which reads [for details see [26]]:

$$\frac{\partial}{\partial y} {}^{QQ}G^T(x, y) + \frac{\partial}{\partial x} {}^{GG}G^T(x, y) = 0. \quad (28)$$

Taking the generic form (26) for  ${}^{AA}G^T(x, y)$ , this differential equation and the symmetry requirements imply the following form of the gluonic addenda (the remaining degree of freedom is fixed by the diagonality of the lowest moments):

$$\Delta {}^{GG}h^T(x, y) = -\frac{2x}{y^2\bar{y}} - \frac{2\bar{x}}{y^2\bar{y}} \ln \bar{x} - \frac{2x}{y\bar{y}^2} \ln y, \quad \Delta {}^{GG}\bar{h}^T(x, y) = \Delta {}^{GG}h^T(\bar{x}, y). \quad (29)$$



The missing diagonal  $^{AA}D^T$  terms are easily obtained in the DGLAP representation by taking the forward limit and comparison with the two-loop results [20], namely,

$$^{AA}D^T(z) = ^{AA}\dot{P}^T(z) - \text{LIM} \left\{ -^{AA}\dot{V}^T \otimes \left( ^{AA}V^{(0)T} + \frac{\beta_0}{2} \mathbb{1} \right) - \left[ ^{AA}g^T \otimes ^{AA}V^{(0)T} \right]_- - \frac{1}{2} G\text{-terms} \right\}. \quad (30)$$

Since the result is expressed in terms of LO splitting functions, we can easily restore the ER-BL representation from them<sup>2</sup>:

$$\begin{aligned} ^{QQ}D^T(x, y) &= -C_F \left( \frac{2}{3} C_F + \frac{5}{6} \beta_0 \right) ^{QQ}v^b(x, y) \\ &\quad - C_F \left( C_F - \frac{C_A}{2} \right) \left\{ -^{QQ}v^a(x, y) - ^{QQ}v^a(\bar{x}, y) + \frac{4}{3} ^{QQ}v^b(x, y) \right\}, \\ ^{GG}D^T(x, y) &= -C_F T_F N_f \left\{ ^{GG}v^a + \frac{2}{3} ^{GG}v^c \right\} (x, y) + \beta_0 C_A \left\{ \frac{3}{8} ^{GG}v^a - \frac{5}{6} ^{GG}v^b + \frac{1}{4} ^{GG}v^c \right\} (x, y) \\ &\quad + C_A^2 \left\{ \frac{13}{8} ^{GG}v^a - \frac{11}{6} ^{GG}v^b + \frac{13}{12} ^{GG}v^c \right\} (x, y), \end{aligned} \quad (31)$$

where the  $b$ -kernel has been defined above in Eq. (19) and the  $a$ - and  $c$ -kernels are given by

$$^{QQ}v^a = \frac{x}{y}, \quad ^{GG}v^a = \frac{x^2}{y^2}, \quad ^{GG}v^c = \frac{x^2}{y^2} (2\bar{x}y + y - x). \quad (32)$$

Contributions concentrated in  $x = y$  will be restored below.

It remains to perform the exclusive convolution. For two non-regularized kernels given by their functions  $f_i(x, y)$  with  $i = \{1, 2\}$  we have

$$(v_1 \otimes v_2)(x, y) = \theta(y - x) (f_1 \otimes f_2)(x, y) + \left\{ \begin{array}{l} x \rightarrow \bar{x} \\ y \rightarrow \bar{y} \end{array} \right\}, \quad (33)$$

where

$$(f_1 \otimes f_2)(x, y) = \int_x^y dz f_1(x, z) f_2(z, y) + \int_y^1 dz f_1(x, z) f_2(\bar{z}, \bar{y}) + \int_0^x dz f_1(\bar{x}, \bar{z}) f_2(z, y). \quad (34)$$

The convolution of two regularized kernels is slightly more involved. Here we define:

$$\left( [v_1]_+ \otimes [v_2]_+ \right) (x, y) = \left[ \theta(y - x) (f_1 \otimes f_2)(x, y) + \left\{ \begin{array}{l} x \rightarrow \bar{x} \\ y \rightarrow \bar{y} \end{array} \right\} \right]_+, \quad (35)$$

where the convolution on the r.h.s. can be decomposed into well-defined integrals

$$\begin{aligned} (f_1 \otimes f_2)(x, y) &= \int_x^y dz \{ [f_1(x, z) - f_1(x, y)] [f_2(z, y) - f_2(x, y)] + [f_1(x, z) - f_1(\bar{z}, \bar{x})] f_2(x, y) \} \\ &\quad + \int_y^1 dz \{ [f_1(x, z) - f_1(x, y)] f_2(\bar{z}, \bar{y}) - [f_1(\bar{z}, \bar{x}) - f_1(x, y)] f_2(x, y) \} \\ &\quad + \int_0^x dz \{ [f_1(\bar{x}, \bar{z}) - f_1(x, y)] [f_2(z, y) - f_2(x, y)] + [f_1(\bar{x}, \bar{z}) - f_1(z, x)] f_2(x, y) \} \\ &\quad - f_1(x, y) f_2(x, y). \end{aligned} \quad (36)$$

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<sup>2</sup>Note that the last term in the  $QQ$  channel was missed in the second article of Ref. [26].

Performing the exclusive convolution in Eq. (20) and adding the diagonal terms given in Eqs. (25)–(27), (29), and (31), the  $QQ$  and  $GG$  transversity kernels in NLO are found to be:

$${}^{QQ}V^{(1)T\pm} = \left[ \theta(y-x) {}^{QQ}\mathcal{F}^{T\pm}(x,y) + C_F^2 \frac{\ln \bar{x} \ln x}{y} + \left\{ \frac{x \rightarrow \bar{x}}{y \rightarrow \bar{y}} \right\} \right]_+ - \frac{1}{2} {}^{QQ}\gamma_0^{(1)T\pm} \delta(x-y), \quad (37)$$

$${}^{GG}V^{(1)T} = \left[ \theta(y-x) {}^{GG}\mathcal{F}^T(x,y) + C_A^2 \left\{ \frac{x(\bar{y}-y)}{y\bar{y}} \ln x + \frac{x+y}{y^2} \ln x \ln \bar{x} \right\} + \left\{ \frac{x \rightarrow \bar{x}}{y \rightarrow \bar{y}} \right\} \right]_+ - \frac{1}{2} {}^{GG}\gamma_1^{(1)T} \delta(x-y), \quad (38)$$

where

$$\begin{aligned} & {}^{QQ}\mathcal{F}^{T\pm}(x,y) \\ &= C_F^2 \left\{ 2 \left( \frac{2}{3} - \zeta(2) \right) {}^{QQ}f^T - {}^{QQ}f^T \left( \frac{3}{2} - \ln \frac{x}{y} \right) \ln \frac{x}{y} - ({}^{QQ}f^T - {}^{QQ}\bar{f}^T) \ln \frac{x}{y} \ln \left( 1 - \frac{x}{y} \right) \right\} \\ & - C_F \left( C_F - \frac{C_A}{2} \right) \left\{ -(1 \mp 1) {}^{QQ}f^a + \frac{4}{3} {}^{QQ}f^T + {}^{QQ}h^T(x,y) \mp {}^{QQ}\bar{h}^T(\bar{x},y) \right\} \\ & - \frac{1}{2} C_F \beta_0 \left( \frac{5}{3} + \ln \frac{x}{y} \right) {}^{QQ}f^T, \end{aligned} \quad (39)$$

$$\begin{aligned} & {}^{GG}\mathcal{F}^T(x,y) \\ &= C_A^2 \left\{ 2 \left( \frac{1}{3} - \zeta(2) \right) {}^{GG}f^T + {}^{GG}f^T \ln^2 \frac{x}{y} - ({}^{GG}f^T - {}^{GG}\bar{f}^T) \ln \frac{x}{y} \ln \left( 1 - \frac{x}{y} \right) \right. \\ & + \frac{13}{4} \left( \frac{1}{3} {}^{GG}f^c + \frac{1}{2} {}^{GG}f^a \right) - \frac{1}{2} {}^{GG}h^T(x,y) - \frac{1}{2} {}^{GG}\bar{h}^T(\bar{x},y) - \Delta {}^{GG}h^T(x,y) \left. \right\} \\ & + \frac{1}{2} C_A \beta_0 \left\{ \frac{3}{4} {}^{GG}f^a - \frac{5}{3} {}^{GG}f^T + \frac{1}{2} {}^{GG}f^c \right\} - C_F N_f T_F \left\{ {}^{GG}f^a + \frac{2}{3} {}^{GG}f^c \right\}. \end{aligned} \quad (40)$$

The contributions concentrated on the diagonal  $x = y$  are fixed by the lowest conformal moment. We have in the  $QQ$  and  $GG$  channels:

$$\begin{aligned} {}^{QQ}\gamma_0^{(1)T\pm} &= \frac{19}{12} C_F^2 - \frac{13}{12} C_F \beta_0 - C_F \left( C_F - \frac{C_A}{2} \right) \left( \frac{19}{3} + (1 \pm 1) [7 - 8\zeta(2) + 4\zeta(3)] \right), \\ {}^{GG}\gamma_1^{(1)T} &= \frac{19}{4} C_A^2 - \frac{9}{4} C_A N_f + \frac{3}{2} C_F N_f. \end{aligned} \quad (41)$$

Finally, we derive the NLO skewed DGLAP kernels in Radyushkin's notation with fraction  $z = \frac{x+\eta}{1+\eta}$  (here  $x$  denotes the momentum fraction in the SPDs) and skewedness  $\zeta = \frac{2\eta}{1+\eta}$ . These kernels govern the evolution of SPDs in the kinematic range of  $\zeta < z < 1$  and  $-1 + \zeta < z < 0$ . We choose this set of variables such that we have the closest resemblance to the usual DGLAP kinematics. In fact for DVCS and vector meson production  $\zeta = x_{Bj}$  up to terms  $\sim \frac{\Delta^2}{Q^2}$ ! The derivation procedure follows that of the forward limit reduction of the extended ER-BL kernels. After the analytical continuation of the ER-BL kernels to the whole  $\{x, y\}$  plane by the replacement of the  $\theta$ -function structure, we derive the kernels in the following way which is suggested by the

non-zero support of the  $\theta$ -functions after the replacement  $x \rightarrow \frac{z}{\zeta}$ ,  $\bar{x} \rightarrow 1 - \frac{z}{\zeta}$ ,  $y \rightarrow \frac{1}{\zeta}$  and  $\bar{y} \rightarrow 1 - \frac{1}{\zeta}$  for the above mentioned kinematical regime:

$$P^T(z, \zeta) = \frac{1}{\zeta} \left\{ {}^{AA}V^T \left( \frac{z}{\zeta}, \frac{1}{\zeta} \right) - {}^{AA}V^T \left( 1 - \frac{z}{\zeta}, 1 - \frac{1}{\zeta} \right) \right\}. \quad (42)$$

Note again that we omitted the additional factor of  $z^{-1}$  in the  $GG$  kernel so as to take into account that in the forward limit, the skewed gluon distribution turns into  $zG(z, Q^2)$  rather than  $G(z, Q^2)$ .

The results for the chiral odd DGLAP kernels in LO, following our above prescription, are:

$${}^{QQ}p^T = \frac{1+z}{\bar{z}} - \frac{1}{\bar{\zeta}}, \quad {}^{GG}p^T = \frac{(1+z)z}{\bar{z}} + \frac{1-\zeta z}{\bar{\zeta}^2} - \frac{1+\zeta+z}{\bar{\zeta}}, \quad (43)$$

and in NLO:

$$\begin{aligned} {}^{QQ}P^{T(1)\pm}(z, \zeta) &= C_F^2 \left\{ \frac{1}{2} \left( {}^{QQ}p^T - \frac{\zeta}{\bar{\zeta}} \right) \ln \left( 1 - \frac{\bar{z}}{\bar{\zeta}} \right) \left[ \ln \left( 1 - \frac{\bar{z}}{\bar{\zeta}} \right) - \frac{3}{2} \right] + \frac{z}{\bar{z}} \ln z \left( \ln z - \frac{3}{2} \right) \right. \\ &\quad \left. - {}^{QQ}p^T \left[ \ln \left( 1 - \frac{\bar{z}}{\bar{\zeta}} \right) \ln \left( \frac{\bar{z}}{\bar{\zeta}} \right) + \ln \bar{z} \ln z \right] + \left( \frac{4}{3} - 2\zeta(2) \right) {}^{QQ}p^T \right\} \\ &\quad - \frac{1}{2} C_F \beta_0 \left\{ \frac{5}{3} {}^{QQ}p^T + \frac{z}{\bar{z}} \ln z + \left( \frac{1}{\bar{z}} - \frac{1}{\bar{\zeta}} \right) \ln \left( 1 - \frac{\bar{z}}{\bar{\zeta}} \right) \right\} \\ &\quad - C_F \left( C_F - \frac{C_A}{2} \right) \left\{ - (1 \mp 1) {}^{QQ}p^a + \frac{4}{3} {}^{QQ}p^T + {}^{QQ}h^T \mp {}^{QQ}\bar{h}^T \right\}, \quad (44) \\ {}^{GG}P^T(z, \zeta) &= C_A^2 \left\{ \frac{1}{2} \ln^2 \left( 1 - \frac{\bar{z}}{\bar{\zeta}} \right) \left( {}^{GG}p^T - \frac{\bar{z} + \zeta(1+z)}{\bar{\zeta}} z + \frac{\bar{z}}{\bar{\zeta}^2} \right) + \frac{13}{4} \left( \frac{1}{3} {}^{GG}p^c + \frac{1}{2} {}^{GG}p^a \right) \right. \\ &\quad \left. - 2 \frac{\zeta}{\bar{\zeta}} \left( \zeta \frac{\bar{z}}{\bar{\zeta}} + \frac{z}{\bar{\zeta}} \ln z - \bar{\zeta} \left( 1 - \frac{\bar{z}}{\bar{\zeta}} \right) \ln \left( 1 - \frac{\bar{z}}{\bar{\zeta}} \right) \right) + \frac{z^2}{\bar{z}} \ln^2 z + \left( \frac{2}{3} - 2\zeta(2) \right) {}^{GG}p^T \right. \\ &\quad \left. - {}^{GG}p^T \left( \ln \left( 1 - \frac{\bar{z}}{\bar{\zeta}} \right) \ln \left( \frac{\bar{z}}{\bar{\zeta}} \right) + \ln \bar{z} \ln z \right) - \frac{1}{2} {}^{GG}h^T - \frac{1}{2} {}^{GG}\bar{h}^T \right\} \\ &\quad + \frac{1}{2} C_A \beta_0 \left\{ \frac{3}{4} {}^{GG}p^a - \frac{5}{3} {}^{GG}p^T + \frac{1}{2} {}^{GG}p^c \right\} - C_F N_f T_F \left\{ {}^{GG}p^a + \frac{2}{3} {}^{GG}p^c \right\}, \quad (45) \end{aligned}$$

where one has to regularize the *whole* kernel with the  $+$ -prescription and add the end-point contribution  $-\frac{1}{2} {}^{QQ}\gamma_0^{(i)T\pm} \delta(1-z)$  and  $-\frac{1}{2} {}^{GG}\gamma_i^{(i)T} \delta(1-z)$  specified earlier. The  ${}^{AA}h^T$  and  ${}^{AA}\bar{h}^T$  functions are given by the equations

$$\begin{aligned} {}^{AA}h^T &= 2 {}^{AA}p^T(z, \zeta) \left[ \ln \zeta \ln \left( 1 - \frac{\bar{z}}{\bar{\zeta}} \right) + \text{Li}_2 \left( 1 - \frac{1}{\zeta} \right) - \text{Li}_2 \left( 1 - \frac{z}{\zeta} \right) - \zeta(2) \right] \\ &\quad + 2 {}^{AA}k_1^T(z, \zeta) \left[ \ln \left( 1 - \frac{\bar{z}}{\bar{\zeta}} \right) \left( \ln \left( \frac{z}{\zeta} \right) - \ln \zeta \right) - 2 \left( \text{Li}_2 \left( 1 - \frac{1}{\zeta} \right) - \text{Li}_2 \left( 1 - \frac{z}{\zeta} \right) \right) \right] \\ {}^{AA}\bar{h}^T &= 2 {}^{AA}p^T(\zeta - z, \zeta) \left[ \ln \zeta \ln \left( 1 - \frac{\bar{z}}{\bar{\zeta}} \right) + \frac{1}{2} \ln \bar{\zeta} \left( \ln \left( \frac{z}{\zeta} \right) + \ln z - \ln \left( \frac{\bar{\zeta}}{\zeta} \right) \right) + \text{Li}_2 \left( 1 - \frac{1}{\zeta} \right) \right. \\ &\quad \left. - \text{Li}_2 \left( 1 - \frac{z}{\zeta} \right) - \zeta(2) - \ln(\bar{\zeta} + z) \left( \ln \left( 1 - \frac{\bar{z}}{\bar{\zeta}} \right) + \ln z \right) - \text{Li}_2(\zeta - z) - \text{Li}_2 \left( -\frac{z}{\bar{\zeta}} \right) \right] \end{aligned}$$

$$+ 2^{AA} k_2^T(z, \zeta) \left[ \ln \left( 1 - \frac{\bar{z}}{\bar{\zeta}} \right) \left( \ln \left( \frac{z}{\zeta} \right) - \ln \zeta \right) - 2 \left( \text{Li}_2 \left( 1 - \frac{1}{\zeta} \right) - \text{Li}_2 \left( 1 - \frac{z}{\zeta} \right) \right) \right]. \quad (46)$$

The remaining skewed kernels appearing in the above formulae read

$$^{QQ}p^a = \frac{\bar{z}}{\bar{\zeta}}, \quad ^{GG}p^a = 2 \frac{z\bar{z}}{\bar{\zeta}^2} - \frac{\zeta}{\bar{\zeta}^2} (1 - z^2), \quad ^{GG}p^c = \frac{\bar{z}^3}{\bar{\zeta}^2}, \quad (47)$$

and the  $^{AA}k_i^T$  were found to be:

$$^{QQ}k_1^T = \frac{z}{\bar{z}}, \quad ^{QQ}k_2^T = -\frac{z}{\bar{\zeta}(\bar{\zeta} + z)}, \quad ^{GG}k_1^T = \frac{z^2}{\bar{z}}, \quad ^{GG}k_2^T = -\frac{z^2}{\bar{\zeta}^2(\bar{\zeta} + z)}. \quad (48)$$

Note, that we have included the addenda of the skewed  $G$ -functions in the single logs and the rational functions of the respective colour structures in the kernels so as to simplify the overall presentation of the results.

## 4 Conclusion.

In conclusion, we have presented a calculation of the two-loop exclusive evolution kernels for skewed chiral odd quark and tensor gluon distributions, thus allowing for NLO analysis of the cross sections where these functions contribute. Our modus operandi is based on the formalism developed previously in Refs. [26] for vector and axial sectors. Due to chirality conservation only the helicity flip gluon SPD shows up in the DVCS cross section. A peculiar azimuthal angle dependence of asymmetries on the nucleon target [9] will give an opportunity to pin down the magnitude of the gluon SPD provided high accuracy data will be available. This will shed some light on the unknown tensor gluon content of the nucleon or higher spin hadrons.

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